Nonlinear Natural Control of an Experimental Beam

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This paper is concerned with an experiment conducted at the NASA Langley Research Center that was designed to control the motion of a beam. The experimental setup consists of a free-free uniform beam acted upon by four electromagnetic force actuators, with the motion being measured by nine displacement sensors. The entire assembly is linked to a digital computer permitting on-line real-time computation of the control forces. The control scheme is based on the independent modal-space control method in conjuction with a nonlinear, on-off control law for the modal forces. The actuator forces are then synthesized from the modal forces by means of a linear transformation, resulting in quantized forces. The sensors' data are processed by modal filters. It is observed that the independent controls are very effective in suppressing the motion of the beam, even though there is about a 50% difference between the actual and the analytically computed natural frequencies.

I. Introduction

An important question in the control of large flexible space structures is whether it is possible to control the structure by means of on-board computers performing real-time computations. To test the ability of various theories to carry out the control task, experiments have been designed by NASA[1,2] and the Jet Propulsion Laboratory.[3,4] This paper reports the results of an experiment conducted at NASA Langley Research Center involving the independent modal-space control (IMSC) method.

The Langley experiment consists of a 12-ft aluminum beam suspended from the ceiling by two cables, so that it behaves like a free-free structure, and involves four electromagnetic force actuators attached to the beam and nine noncontacting displacement sensors. The sensors' data are transmitted to a CDC Cyber 175 digital computer, which generates control forces in discrete time. These force commands are transmitted to an EAI 680 analog computer that, through a high-gain electrical circuit, generates the current commands to the actuators.

Theoretically, any type of control law can be used to suppress the motion of a flexible structure. In practice, however, there is a constraint on the type of control laws in the time required between displacement sensing and the application of the control forces. Indeed, the entire process is carried out in discrete time, and there is a lag of one sampling period between the displacement sensing and and the application of the control forces.[5] This implies that the entire control cycle (consisting of the displacement sensing, processing the data on the computers, and application of the feedback control forces by the actuators) must be smaller than the sampling period. On the other hand, there is a limit on the length of the sampling period. Indeed, if the sampling period is too large, then the lag between the displacement sensing and the application of the forces by the actuators can cause instability, so that the sampling period should be made as small as possible. This lag of one sampling period can be eliminated if the control forces are computed in a very small fraction of the sampling time and reset to their new values in the beginning of the sampling interval. This results in improved stability characteristics of the control, but places more restrictions on the number of computations that can be performed during the control cycle. Hence, a control algorithm capable of generating the actuator forces from the sensors' data in minimum time is highly desirable. Note that, because of the compensating circuits involved in the experimental setup, which would not be needed in an actual space environment, the lag of one sampling period could not be eliminated for the Langley beam experiment.

In one sense or another, most methods for the control of flexible structures are modal control ones.[6-10] The idea behind modal control is to control the motion of a structure by controlling its modes. Modal control is dictated by the fact that a structure is a distributed-parameter system and one would encounter enormous computational difficulties in deriving control gains depending on the spatial position. One approach to modal control is to consider only a finite number of modes, generally the lower ones. Then, expressing the motion as a superposition of these modes, the partial differential equation of motion is replaced by a set of ordinary differential modal equations. In general, the modal equations are coupled by the feedback forces.[7] In the special case of the IMSC method, the modal feedback forces are independent, so that the closed-loop equations of motion are decoupled.[8-10] Hence, by analogy with the term "natural coordinates" used in vibration analysis to denote uncoupled coordinates, independent controls are referred to as *natural controls*. Then, the actuator forces are synthesized from the independent modal forces by a simple linear transformation.

The fact that in IMSC the feedback forces are designed independently for each mode permits an enormous computational saving in the generation of the actuators' forces from the sensors' data, so that the control cycle is minimal. In turn, this permits a relatively small sampling period, thus obviating the instability problem that would be caused by a large sampling period. Moreover, because the control problem is reduced to that of controlling a set of independent section-order systems, natural control can accommodate a large variety of control laws, including nonlinear controls. To demonstrate this, in the present study nonlinear control laws

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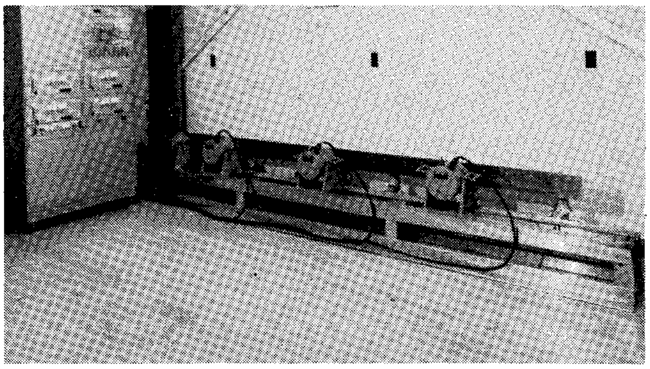

Fig. 1 The Langley experimental beam setup.

are used in the form of on-off modal controls. Because the actual controls are linear combinations of the modal controls, the actual controls are quantized.

There exist two major procedures for processing the sensors' data to obtain the modal coordinates so as to permit design of modal control forces. The first relies on the use of observers.[11] It has been shown, however, that the use of observers leads to observation spillover from the uncontrolled modes capable of destabilizing these modes.[7] The second consists of implementing modal filters.[9] Modal filters can eliminate observation spillover, provided an adequate number of measurements is used. They also require less computational effort for implementation than observers. Observers and modal filters are not mutually exclusive and one can use modal filters for modal displacements in conjunction with modal observers to recover the modal velocities. In the present study, modal filters are used exclusively. In the Langley beam experiment, the four lowest modes—the translational and rotational rigid body modes and the two lowest flexible modes—were controlled using a nonlinear on-off modal control law based on IMSC. Natural control was very effective in suppressing the motion of the beam, damping out the motion in a very short period of time, in spite of the fact that the actual frequencies were about 50% higher than the frequencies determined by an analytical approach. This demonstrated the robustness of the IMSC method using a nonlinear control law in an ad hoc manner. The robustness of the IMSC method for linear control has been demonstrated earlier.[12]

II. Experimental Setup

The experimental apparatus is shown in Figs. 1 and 2, and it consists of a 12 ft 6061 aluminum beam suspended from the ceiling by two lightweight 5 ft cables.[1] The beam weighs 16.2 lb and has a cross section of $6 \times 3/16$ in. The material properties of the beam are $E = 10^7$ psi, $\rho = 0.1$ lbm/in. (Ref. 3). It was assumed that the beam acts like a free-free structure, so that the beam was modeled and the control forces were designed ignoring the cables.

Four electromagnetic force actuators and nine noncontacting displacement sensors were used. The actuators have a stroke of 1 in. and a maximum force output of 50 lbf. They are attached to the beam with spring steel flexures that permit transverse vibration in the horizontal plane while suppressing torsional vibration. The flexures are attached to strain gage load cells hard mounted to the beam. An additional purpose of the suspension cables is to maintain a horizontal loading condition by preventing droop of the end of the actuator attachment assembly. On the side of the beam, opposite the side of the actuators, are noncontacting deflection sensors with a range of 2 in. The resolution of the sensors is 0.001 in. The nine sensors can be placed at arbitrary stations along the beam.

The components mentioned above are interfaced to the NASA Langley (LaRC) Cyber 175 real-time computer system.

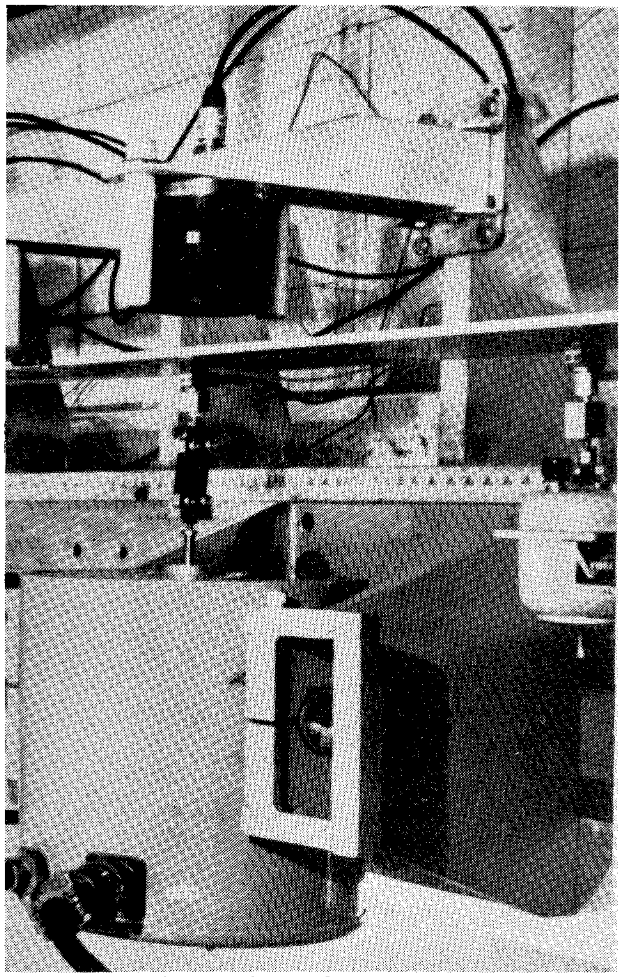

Fig. 2 Closeup of sensor-beam-actuator assembly.

A high-gain analog electrical circuit assures that commanded forces from the control system resident on the Cyber computer are realized at the load cells on the beam. For this reason, the force commands form the Cyber computer are sent to the EAI 680 analog computer that, through the use of the circuit, generates the current commanded to the actuators.[1] The maximum commanded load from the Cyber is set to 10 lbf, whereas the maximum value that the Cyber can read from the load cells is 20 lbf. Four of the sensors are collocated with the actuators to compensate for actuator nonlinearities. Details of the actuator compensation circuit can be found in Ref. 1. The sampling time was chosen as 1/64 s.

III. Modal Equations of Motion

The equation of motion for a self-adjoint distributed-parameter system can be written in the form of the partial differential equation (Ref. 13, Sec. 5-4).

$$Lu(P,t) + M(P)\partial^2 u(P,t)/\partial t^2 = f(P,t) \tag{1}$$

which must be satisfied at every point P of the domain D, where $u(P,t)$ is the displacement of point P, L a linear differential self-adjoint operator of order $2p$, $M(P)$ the distributed mass, and $f(P,t)$ the distributed controls. The displacement $u(P,t)$ is subject to the boundary conditions $B_i u(P,t) = 0$ $(i = 1,2,...,p)$, where B_i are linear differential operators. The solution of the associated eigenvalue problem consists of a denumerably infinite set of eigenvalues λ_r and associated eigenfunctions ϕ_r $(r = 1,2,...)$. The eigenvalues are related to the natural frequencies ω_r by $\lambda_r = \omega_r^2$ $(r = 1,2,...)$.

Note that frequencies corresponding to rigid-body modes are zero. Because L is self-adjoint, the eigenfunctions possess the orthogonality property and they can be normalized so as to satisfy

$$\int_D M\phi_r\phi_s \mathrm{d}D=\delta_{rs},\quad \int_D \phi_s L\phi_r \mathrm{d}D=\lambda_r\delta_{rs},\quad r,s=1,2,\ldots$$

where δ_{rs} is the Kronecker delta.

Using the expansion theorem[13]

$$u(P,t)=\sum_{r=1}^{\infty}\phi_r(P)u_r(t) \tag{2}$$

where $u_r(t)$ are *modal coordinates*, Eq. (1) can be replaced by the infinite set of ordinary differential equations

$$\ddot{u}_r(t)+\omega_r^2u_r(t)=f_r(t),\qquad r=1,2,\ldots \tag{3}$$

known as *modal equations*, in which

$$f_r(t)=\int_D \phi_r(P)f(P,t)\mathrm{d}D,\qquad r=1,2,\ldots \tag{4}$$

are *modal control forces*.

Equations (3) have the appearance of an infinite set of independent second-order differential equations and, in the absence of feedback control forces, they indeed are. If feedback control forces are present, however, and the modal feedback control force $f_r(t)$ depends on all the modal coordinates and velocities, Eqs. (3) are coupled through the feedback controls. Hence, in this general case Eqs. (3) are externally coupled, so that the equations are not independent. In the special case in which f_r depends on u_r and $\dot{u}_r$ alone, $f_r=f_r(u_r,\dot{u}_r)$ $(r=1,2,\ldots)$, Eqs. (4) become both internally and externally decoupled. This is the essence of the independent modal-space control method.[8-10] Because the modal equations are independent, by analogy with the term "natural coordinates" used in vibration analysis to denote uncoupled coordinates, the independent modal-space control is referred to as *natural control*.[10] Natural control permits both linear and nonlinear control, and the method has been shown to be very effective in controlling distributed systems.[10]

Natural control can be implemented by means of discrete or distributed actuators and sensors. We consider here the problem of controlling n modes using m discrete actuators. The discrete actuator forces can be treated as distributed by writing

$$f(P,t)=\sum_{j=1}^{m}F_j(t)\delta(P-P_j) \tag{5}$$

Introducing Eq. (5) into Eqs. (4), we obtain

$$f_r(t)=\sum_{j=1}^{m}\int_D \phi_r(P)F_j(t)\delta(P-P_j)\mathrm{d}D$$
$$=\sum_{j=1}^{m}\phi_r(P_j)F_j(t),\qquad r=1,2,\ldots,n \tag{6}$$

Introducing the vectors

$$f(t)=[f_1(t)\quad f_2(t)\ldots f_n(t)]^T \tag{7a}$$

$$F(t)=[F_1(t)\quad F_2(t)\ldots F_m(t)]^T \tag{7b}$$

and the $n\times m$ matrix

$$B=[B_{rj}]=[\phi_r(P_j)],\qquad r=1,2,\ldots,n;\qquad j=1,2,\ldots,m \tag{8}$$

we can write

$$f(t)=BF(t) \tag{9}$$

In the IMSC method the modal control vector is determined first. Then, the actual controls can be synthesized from the modal controls by

$$F(t)=B^{-1}f(t) \tag{10}$$

which requires B to be a square and nonsingular matrix. Hence, implementation of natural control by discrete actuators requires that the number of actuators be equal to the number of controlled modes.

The effect of having discrete actuators is that some of the control energy is pumped into the uncontrolled modes, creating control spillover. This is typical of any control scheme.

For feedback controls, one must extract the modal displacements and velocities associated with the controlled modes from the system output. To this end, we can make use of the second part of the expansion theorem[13] and write

$$u_r(t)=\int_D M(P)\phi_r(P)u(P,t)\mathrm{d}D,\qquad r=1,2,\ldots \tag{11a}$$

$$\dot{u}_r(t)=\int_D M(P)\phi_r(P)\dot{u}(P,t)\mathrm{d}D,\qquad r=1,2,\ldots \tag{11b}$$

Equations (11) can be regarded as *modal filters*. They permit the extraction of the modal displacement $u_r(t)$ and modal velocity $\dot{u}_r(t)$ from distributed measurements of the actual displacement $u(P,t)$ and actual velocity $\dot{u}(P,t)$ at every point P of the domain D and at all times t. Hence, if modal filters are used, then all the system modes are observable and can be regarded as modeled, so that *control of the actual distributed system is possible*, although in practice control of a finite number of modes is often sufficient. Having $u_r(t)$ and $\dot{u}_r(t)$, one can generate the modal controls $f_r(t)$.

The modal filters described by Eqs. (11) require distributed measurements of $u(P,t)$ and $\dot{u}(P,t)$ and integration along the spatial domain. However, the present state-of-the-art permits only discrete measurements. It is shown in Ref. 9 that it is possible to implement modal filters by using only a finite number of discrete sensors and by spatially interpolating the sensors' output to obtain displacement and velocity profiles. The interpolation functions can be chosen from the finite element method and the integrations can be performed as off-line computations.[9] The displacement and velocity profiles thus generated can then be used to extract the modal displacements and velocities associated with the controlled modes. Hence, a finite number of sensors can estimate a given number of modal displacements and velocities accurately; furthermore, it is relatively easy to determine this number. The estimated modal coordinates become[14]

$$\hat{u}_r(t)=\sum_{i=1}^{s}I_{ir}^T v_i(t),\qquad \hat{\dot{u}}_r(t)=\sum_{i=1}^{s}I_{ir}^T\dot{v}_i(t),\quad r=1,2,\ldots,n \tag{12}$$

where $\hat{u}_r$ and $\hat{\dot{u}}_r$ are the estimated modal displacements and velocities, respectively, s the number of subdomains, and $v_i(t)$ a vector of displacement measurements and $\dot{v}_i(t)$ a vector of velocity measurements at the boundaries of the ith subdomain. In addition,

$$I_{ir}=\int_{D_i} M(P)\phi_r(P)L\mathrm{d}D,\quad i=1,2,\ldots,s;\quad r=1,2,\ldots,n \tag{13}$$

where L is a vector of interpolation functions from the finite element method. It is clear from Eqs. (13) that the integrations can be performed as off-line computations.

The quantities I_{ir} can be rearranged so as to permit writing

$$\hat{u}_r(t) = \sum_{j=1}^{k} C_{rj} y_j(t), \quad \dot{\hat{u}}_r(t) = \sum_{j=1}^{k} C_{rj} \dot{y}_j(t), \quad r=1,2,\ldots,n \tag{14}$$

where

$$y_j(t) = u(P_j, t), \qquad j=1,2,\ldots,k \tag{15}$$

in which P_j denote the sensors' locations. Introducing the vectors

$$\hat{u}(t) = [\hat{u}_1(t) \quad \hat{u}_2(t) \ldots \hat{u}_n(t)]^T$$

$$\dot{\hat{u}}(t) = [\dot{\hat{u}}_1(t) \quad \dot{\hat{u}}_2(t) \ldots \dot{\hat{u}}_n(t)]^T$$

$$y(t) = [y_1(t) \quad y_2(t) \ldots y_k(t)]^T$$

$$\dot{y}(t) = [\dot{y}_1(t) \quad \dot{y}_2(t) \ldots \dot{y}_k(t)]^T \tag{16}$$

and the $n \times k$ matrix $C = [C_{rj}]$, we can write

$$\hat{u}(t) = Cy(t), \qquad \dot{\hat{u}}(t) = C\dot{y}(t) \tag{17a,b}$$

The manner in which C is assembled from I_{ir} depends on the nature of the interpolation functions.

IV. The Langley Beam Experiment

The parameters associated with the beam are given in Sec. II. The mass and stiffness distributions are $M(x) = M = 2.911 \times 10^{-4}$ lb·s²/in.², $EI(x) = EI = 3296$ lb.·in.², $\ell = 144$ in., $L = EI\, d^4/dx^4$, $0 < x < \ell$, where x is the spatial variable. The eigensolution of the free-free beam can be obtained in closed form.[13] The eigenfunctions are

$$\phi_1(x) = 1, \qquad \phi_2(x) = x - \ell/2$$

$$\phi_r(x) = (\cos\beta_r\ell - \cosh\beta_r\ell)(\sin\beta_r x + \sinh\beta_r x)$$
$$- (\sin\beta_r\ell - \sinh\beta_r\ell)(\cos\beta_r x + \cosh\beta_r x), \quad r=3,4,\ldots \tag{18}$$

where $\beta_r = (\omega_r^2 M/EI)^{1/4}$ $(r=3,4,\ldots)$. Note that in Eqs. (18) the eigenfunctions are not in normalized form. They were normalized later when designing the control forces. The natural frequencies ω_r are

$$\omega_1 = \omega_2 = 0 \text{ rad/s}, \quad \omega_3 = 11.480 \text{ rad/s}, \quad \omega_4 = 31.645 \text{ rad/s},$$

$$\omega_5 = 62.036 \text{ rad/s}, \quad \omega_6 = 102.55 \text{ rad/s}, \quad \omega_7 = 153.19 \text{ rad/s}$$

The zero frequencies correspond to the translational and rotational rigid-body modes. The nonzero natural frequencies are determined by solving a transcendental characteristic equation numerically. The lowest four modes were controlled, using IMSC in conjunction with a nonlinear on-off control law. The on-off controls provide for a deadband region on the assumption that some small oscillations can be tolerated. Implementation of the on-off controls makes use of switching curves in the phase plane. Note that the on-off controls are modal controls. The actual control forces are quantized, i.e., they represent linear combinations of on-off functions.

The actuators were located at 0.5, 2.5, 6.5, and 9.5 ft. The modal control forces were selected as follows:

1) For rigid-body modes,

$$\eta_r = |\hat{u}_r| + |\dot{\hat{u}}_r|/c_r$$

If $\eta_r < d_r$, then $f_r = 0$. If $\eta_r > d_r$ and in addition

a) $\hat{u}_r > 0$ and $\dot{\hat{u}}_r > 0$, or $\hat{u}_r > 0 > \dot{\hat{u}}_r$ and $|\dot{\hat{u}}_r| < \epsilon_r$, then $f_r = -k_r$.

b) $\hat{u}_r < 0$ and $\dot{\hat{u}}_r < 0$, or $\hat{u}_r < 0 < \dot{\hat{u}}_r$ and $|\dot{\hat{u}}_r| < \epsilon_r$, then $f_r = k_r$. $\qquad r=1,2 \qquad$ (19)

2) For elastic modes,

$$f_r = \begin{cases} -k_r, & \hat{u}_r \geq d_r \\ 0, & |\hat{u}_r| < d_r, \\ k_r, & \hat{u}_r \leq -d_r \end{cases} \qquad r=3,4 \tag{20}$$

where d_r is the magnitude of the deadband region, ϵ_r the threshold velocity, c_r a weighting factor, and k_r the magnitude of the modal control force. The values selected were

$$k_1 = 0.3, \quad d_1 = 0.002, \quad \epsilon_1 = 0.01$$

$$k_2 = 0.30 \quad d_2 = 0.002, \quad \epsilon_2 = 0.02$$

$$k_3 = 0.11, \quad d_3 = 0.006, \quad c_1 = 2.00$$

$$k_4 = 0.10, \quad d_4 = 0.008, \quad c_2 = 2.00$$

The actuator force vector $F(t)$ was synthesized from the modal control force vector $f(t)$ according to Eq. (10). The nine displacement sensors were located at 0, 0.5, 2.5, 4.0, 6.5, 7.5, 9.5, 11.0, and 12 ft. The actuators were collocated with four of the sensors, so that the nonlinearities in the actuator performance could be compensated.[1] The interpolation functions were selected using concepts from the finite element methods. The beam was divided into four elements such that there were sensors at both ends of each element (external nodes) and one sensor inside the element (internal node). From Eq. (11a) we can write

$$\hat{u}_r(t) = \sum_{j=1}^{4} \int_{x_{2j-1}^s}^{x_{2j+1}^s} M(x)\phi_r(x)\hat{u}_j(x,t)\,dx \tag{21}$$

where x_{2j-1}^s and x_{2j+1}^s $(j=1,2,3,4)$ denote the boundaries of the jth element. The length of each element is

$$h_j = x_{2j+1}^s - x_{2j-1}^s, \qquad j=1,2,\ldots,s; \qquad s=4 \tag{22}$$

To carry out integrations involving the interpolation functions, we define the local coordinate ζ, related to the global coordinate x by

$$\zeta = (x_{2j+1}^s - x)/h_j, \qquad j=1,2,3,4, \qquad 0 < \zeta < 1 \tag{23}$$

The displacement in the jth element can be approximated by

$$\hat{u}_j(\zeta,t) = \sum_{i=1}^{3} L_i x_{2j-i}^s \tag{24}$$

where L_i are interpolation functions having the form[15]

$$L_1 = -\frac{a_j}{1-a_j}\zeta + \frac{1}{1-a_j}\zeta^2$$

$$L_2 = \frac{1}{a_j(1-a_j)}\zeta(1-\zeta), \qquad j=1,2,\ldots,s; \qquad s=4 \tag{25}$$

$$L_3 = 1 - \left(1 + \frac{1}{a_j}\right)\zeta + \frac{1}{a_j}\zeta^2$$

where $a_j = (x_{2j}^s - x_{2j-1}^s)/h_j$. Substituting Eqs. (22-25) into Eqs. (21), we obtain

$$I_{jr} = h_j \int_0^1 M\phi_r(x_{2j+1}^s - \zeta h_j) L(\zeta)\,d\zeta \tag{26}$$

where

$$L(\zeta) = [L_1 \; L_2 \; L_3]^T \tag{27}$$

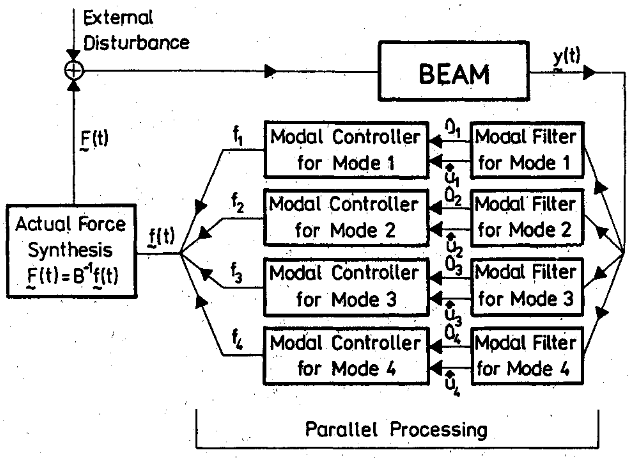

Fig. 3 Block diagram representation of natural control.

The matrix C can be constructed from I_{jr} by means of the relation

$$C_{rp} = \sum_{j=1}^{s} \sum_{\ell=1}^{3} I_{jr\ell}\delta_{2r+\ell-2,p}$$

$$r=1,2,\ldots,n, \quad n=4; \quad p=1,2,\ldots,k; \quad k=9 \qquad (28)$$

where ℓ denotes the interpolation function and j the element number.

The process of obtaining the entries of the matrix C may appear unduly complicated, but is is not much different from that for deriving the mass and stiffness matrices in a finite element analysis. Note that all the computations can be carried out off-line before the control process begins, so that there is no real-time computational burden.

Because velocity measurements were not available, the modal velocities were estimated from

$$\hat{\dot{u}}_r(t) = \frac{\hat{u}_r(t) - \hat{u}_r(t-\Delta t)}{\Delta t}, \qquad r=1,2,\ldots,n; \qquad n=4 \quad (29)$$

where Δt is the sampling period. It was chosen as 1/64 s.

The steps involved in the real-time, on-line control can be summarized as follows:

1) Read the sensors' output $y(t)$.

2) Convert the sensors output into modal displacements by using Eq. (17a). This step requires $n \times k$ multiplications, so that 36 multiplications are needed for the beam under consideration.

3) Estimate the modal velocities using Eq. (29). This step requires $n=4$ multiplications.

4) Determine the modal control forces using Eqs. (19) and (20). This step requires two "IF" statements per controlled mode, or 8 "IF" statements in the case of our experiment.

5) Synthesize the actual control forces $F(t)$ by using Eq. (10). This step requires $n^2=16$ multiplications.

Figure 3 describes the control cycle in block diagram form. The total computational effort per sampling period is $n(k+n+1)$ multiplications and $2n$ "IF" statements. For the Langley beam experiment 56 multiplications and 8 "IF" statements have to be performed during each sampling period. This is an extremely small number of computations. Hence, in a space environment, in which circuits compensating for effects present in a laboratory environment are not needed, the entire control task can be carried out by microcomputers. In addition, it is clear from the above that natural control permits parallel processing to compute the control forces, so that the computational time per cycle can be reduced even more.

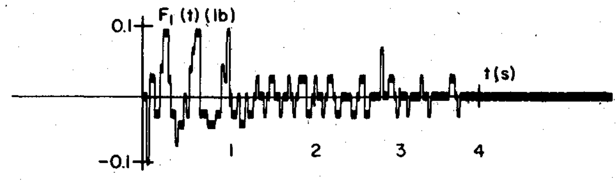

Fig. 4 Actuator force at $x=0.5$ ft.

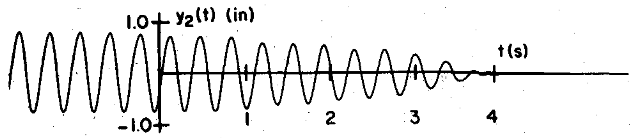

Fig. 5 Sensor output at $x=0.5$ ft.

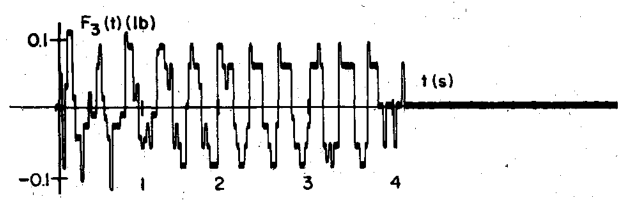

Fig. 6 Actuator force at $x=6.5$ ft.

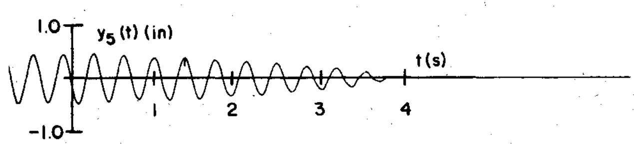

Fig. 7 Sensor output at $x=6.5$ ft.

Control of the beam was carried out for a disturbance in the form of an initial unit impulse. Figures 4-7 show the sensors' output and actuator forces. It is clear that IMSC performed extremely well in controlling the motion of the beam. The control forces exerted by the actuators seldom exceeded 0.1 lbf and all displacements decayed within nine cycles of closed-loop oscillation. This may appear to be a long time to damp out the motion, but it should be noted that the beam admits rigid-body motion, and rigid-body modes have infinite period. Note that, because the modal controls are on-off with bounded amplitudes by design, the magnitude of the control effort is bounded, so that the control requires a certain number of cycles. In designing modal control forces for rigid-body modes, one must ensure that the applied impulse is not too large.

A comparison of Figs. 5 and 7 indicates that the displacement amplitude is higher in Fig. 5. This is because Fig. 5 displays the displacement of a point close to the end of the beam, and the contribution of the rigid-body rotation to this displacement is much larger than the contribution to the displacement of a point close to the center of the beam, where the latter is plotted in Fig. 7. Note that the left sides of Figs. 5 and 7 represent the uncontrolled response, before the controls go into action.

It was observed that the rigid-body modes and the first flexible mode dominate the behavior of the beam and that the second flexible mode contributes very little to the motion of the beam. Control spillover into the uncontrolled modes was negligible. This is because all the dominant modes were controlled. From previous computer simulations and system response, it is clear that modal filters using nine sensors were capable of producing very accurate estimates of the modal coordinates and velocities for the controlled modes.

It was also observed that the actual frequencies of the flexible modes are about 50% higher than the frequencies computed by an eigenvalue analysis based on the assumption that the beam was free-free. This difference is very likely due to two reasons. One reason lies in the lack of compensation of the actuator dynamics. The actuators are attached to the beam and each has a restoring spring on its shaft, which increases the stiffness of the beam. Another reason is that the assumption that the beam behaves like a free-free structure may not be good enough, as it ignores the effects of the supporting cables. These cables give rise to restoring forces, thus raising the natural frequencies. In spite of the fact that the model contains serious parameter errors, the control scheme worked very well. This result is similar to the result obtained in Ref. 12, in which it is shown that, when IMSC is used in conjunction with modal filters, errors in the system parameters cannot destabilize the actual distributed system. Hence, the results of this experiment may be regarded as evidence of robustness of the IMSC method even for nonlinear controls.

Finally, it should be noted that even though the number of computations per control cycle is very small, because of the compensating circuits required by the experimental setup, the control forces were computed and applied after a delay of one sampling period. This delay resulted in phase angles of 10 and 30 deg for the controlled flexible modes. It was observed that theses phase angles did not affect the control system performance perceptibly. It should be mentioned that when a sampling period of 1/32 s was used the second flexible mode became unstable, as the phase lag became excessively large. In addition, Eq. (29) loses its accuracy for the higher modes as the sampling period is lowered, because the number of measurements during one period is reduced.

V. Conclusions

Results of a recent experiment conducted at NASA Langley Research Center demonstrate the effectiveness of the independent modal-space control method in controlling the motion of a free-free beam using nonlinear on-off modal control laws. Note that on-off modal controls translate into quantized actual controls. The control forces are implemented by four electromagnetic force actuators in conjunction with nine displacement sensors. It is observed that the natural control scheme is very effective in suppressing the motion of the beam, in spite of the fact that there are serious parameter errors in the model. It is also shown that the controls require a minimal amount of computational effort, so that in a space environment the entire control scheme can be implemented with microprocessors.

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